Singapore Examinations and Assessment Board

Cambridge Assessment International Education

Singapore–Cambridge General Certificate of Education Ordinary Level (2025)

Additional Mathematics (Syllabus 4049)

CONTENTS

INTRODUCTION

The syllabus prepares students adequately for A-Level H2 Mathematics, where a strong foundation in algebraic manipulation skills and mathematical reasoning skills are required. The content is organised into three strands, namely, Algebra, Geometry and Trigonometry, and Calculus. Besides conceptual understanding and skill proficiency explicated in the content strands, important mathematical processes such as reasoning, communication and application (including the use of models) are also emphasised and assessed. The O-Level Additional Mathematics syllabus assumes knowledge of O-Level Mathematics.

AIMS

The O-Level Additional Mathematics syllabus aims to enable students who have an aptitude and interest in mathematics to:

- acquire mathematical concepts and skills for higher studies in mathematics and to support learning in the other subjects, with emphasis in the sciences, but not limited to the sciences

- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem-solving

- connect ideas within mathematics and between mathematics and the sciences through applications of mathematics; and

- appreciate the abstract nature and power of mathematics.

ASSESSMENT OBJECTIVES

The assessment will test candidates' abilities to:

AO1 **Use and apply standard techniques**
- recall and use facts, terminology and notation
- read and use information directly from tables, graphs, diagrams and texts
- carry out routine mathematical procedures

AO2 **Solve problems in a variety of contexts**
- interpret information to identify the relevant mathematics concept, rule or formula to use
- translate information from one form to another
- make and use connections across topics/subtopics
- formulate problems into mathematical terms
- analyse and select relevant information and apply appropriate mathematical techniques to solve problems
- interpret results in the context of a given problem

AO3 **Reason and communicate mathematically**
- justify mathematical statements
- provide explanation in the context of a given problem
- write mathematical arguments and proofs

Approximate weightings for the assessment objectives are as follows:

AO1	35%
AO2	50%
AO3	15%

SCHEME OF ASSESSMENT

Paper	Duration	Description	Marks	Weighting
Paper 1	2 hours 15 minutes	There will be 12 – 14 questions of varying marks and lengths, up to 10 marks per question. Candidates are required to answer **ALL** questions.	90	50%
Paper 2	2 hours 15 minutes	There will be 9 – 11 questions of varying marks and lengths, up to 12 marks per question. Candidates are required to answer **ALL** questions.	90	50%

NOTES

1. Omission of essential working will result in loss of marks.

2. Relevant mathematical formulae will be provided for candidates.

3. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. In questions which explicitly require an answer to be shown to be correct to a specific accuracy, the answer must be first shown to a higher degree of accuracy.

4. SI units will be used in questions involving mass and measures.
   Both the 12-hour and 24-hour clock may be used for quoting times of the day. In the 24-hour clock, for example, 3.15 a.m. will be denoted by 03 15; 3.15 p.m. by 15 15.

5. Candidates are expected to be familiar with the solidus notation for the expression of compound units, e.g. 5 m/s for 5 metres per second.

6. Unless the question requires the answer in terms of π, the calculator value for π or π = 3.142 should be used.

7. Spaces will be provided in each question paper for working and answers.

USE OF CALCULATORS

An approved calculator may be used in **both** Paper 1 and Paper 2.

SUBJECT CONTENT

Knowledge of the content of O-Level Mathematics syllabus is assumed in the syllabus below and will not be tested directly, but it may be required indirectly in response to questions on other topics.

Topic/Sub-topics		Content
ALGEBRA		
A1	**Quadratic functions**	• Finding the maximum or minimum value of a quadratic function using the method of completing the square • Conditions for $y = ax^2 + bx + c$ to be always positive (or always negative) • Using quadratic functions as models
A2	**Equations and inequalities**	• Conditions for a quadratic equation to have: (i) two real roots (ii) two equal roots (iii) no real roots and related conditions for a given line to: (i) intersect a given curve (ii) be a tangent to a given curve (iii) not intersect a given curve • Solving simultaneous equations in two variables by substitution, with one of the equations being a linear equation • Solving quadratic inequalities, and representing the solution on the number line
A3	**Surds**	• Four operations on surds, including rationalising the denominator • Solving equations involving surds
A4	**Polynomials and partial fractions**	• Multiplication and division of polynomials • Use of remainder and factor theorems, including factorising polynomials and solving cubic equations • Use of: - $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ - $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ • Partial fractions with cases where the denominator is no more complicated than: - $(ax + b)(cx + d)$ - $(ax + b)(cx + d)^2$ - $(ax + b)(x^2 + c^2)$
A5	**Binomial expansions**	• Use of the Binomial Theorem for positive integer n • Use of the notations $n!$ and $\binom{n}{r}$ • Use of the general term $\binom{n}{r} a^{n-r} b^r$, $0 \leqslant r \leqslant n$ (knowledge of the greatest term and properties of the coefficients is not required)

<table>
<tr><th colspan="2">Topic/Sub-topics</th><th>Content</th></tr>
<tr><td>A6</td><td>Exponential and logarithmic functions</td><td>• Exponential and logarithmic functions a^x, e^x, $\log_a x$, $\ln x$ and their graphs, including
– laws of logarithms
– equivalence of $y = a^x$ and $x = \log_a y$
– change of base of logarithms
• Simplifying expressions and solving simple equations involving exponential and logarithmic functions
• Using exponential and logarithmic functions as models</td></tr>
<tr><td colspan="3">GEOMETRY AND TRIGONOMETRY</td></tr>
<tr><td>G1</td><td>Trigonometric functions, identities and equations</td><td>• Six trigonometric functions for angles of any magnitude (in degrees or radians)
• Principal values of $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$
• Exact values of the trigonometric functions for special angles (30°, 45°, 60°) or $(\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3})$
• Amplitude, periodicity and symmetries related to sine and cosine functions
• Graphs of $y = a\sin(bx) + c$, $y = a\sin\left(\frac{x}{b}\right) + c$, $y = a\cos(bx) + c$, $y = a\cos\left(\frac{x}{b}\right) + c$ and $y = a\tan(bx)$, where a is real, b is a positive integer and c is an integer.
• Use of:
– $\frac{\sin A}{\cos A} = \tan A$, $\frac{\cos A}{\sin A} = \cot A$, $\sin^2 A + \cos^2 A = 1$, $\sec^2 A = 1 + \tan^2 A$, $\mathrm{cosec}^2 A = 1 + \cot^2 A$
– the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$
– the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$
– the expression of $a\cos\theta + b\sin\theta$ in the form $R\cos(\theta \pm \alpha)$ or $R\sin(\theta \pm \alpha)$
• Simplification of trigonometric expressions
• Solution of simple trigonometric equations in a given interval (excluding general solution)
• Proofs of simple trigonometric identities
• Using trigonometric functions as models</td></tr>
<tr><td>G2</td><td>Coordinate geometry in two dimensions</td><td>• Condition for two lines to be parallel or perpendicular
• Midpoint of line segment
• Area of rectilinear figure
• Coordinate geometry of circles in the form:
– $(x-a)^2 + (y-b)^2 = r^2$
– $x^2 + y^2 + 2gx + 2fy + c = 0$
(excluding problems involving two circles)
• Transformation of given relationships, including $y = ax^n$ and $y = kb^x$, to linear form to determine the unknown constants from a straight line graph</td></tr>
</table>

Topic/Sub-topics		Content
G3	**Proofs in plane geometry**	• Use of: - properties of parallel lines cut by a transversal, perpendicular and angle bisectors, triangles, special quadrilaterals and circles∗ - congruent and similar triangles∗ - midpoint theorem - tangent-chord theorem (alternate segment theorem)
CALCULUS		
C1	**Differentiation and integration**	• Derivative of $\mathrm{f}(x)$ as the gradient of the tangent to the graph of $y = \mathrm{f}(x)$ at a point • Derivative as rate of change • Use of standard notations $\mathrm{f}'(x), \mathrm{f}''(x), \frac{\mathrm{d}y}{\mathrm{d}x}, \frac{\mathrm{d}^2y}{\mathrm{d}x^2}[=\frac{\mathrm{d}}{\mathrm{d}x}(\frac{\mathrm{d}y}{\mathrm{d}x})]$ • Derivatives of x^n, for any rational n, $\sin x$, $\cos x$, $\tan x$, e^x, and $\ln x$ together with constant multiples, sums and differences • Derivatives of products and quotients of functions • Use of Chain Rule • Increasing and decreasing functions • Stationary points (maximum and minimum turning points and stationary points of inflexion) • Use of second derivative test to discriminate between maxima and minima • Apply differentiation to gradients, tangents and normals, connected rates of change and maxima and minima problems • Integration as the reverse of differentiation • Integration of x^n for any rational n, $\sin x$, $\cos x$, $\sec^2 x$ and e^x, together with constant multiples, sums and differences • Integration of $(ax+b)^n$ for any rational n, $\sin(ax+b)$, $\cos(ax+b)$ and $\mathrm{e}^{(ax+b)}$ • Definite integral as area under a curve • Evaluation of definite integrals • Finding the area of a region bounded by a curve and line(s) (excluding area of region between 2 curves) • Finding areas of regions below the x-axis • Application of differentiation and integration to problems involving displacement, velocity and acceleration of a particle moving in a straight line

∗ These are properties learnt in O-Level Mathematics

MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}bc \sin A$$

MATHEMATICAL NOTATION

The list which follows summarises the notation used in Cambridge's Mathematics examinations. Although primarily directed towards A-Level, the list also applies, where relevant, to examinations at all other levels.

1. *Set Notation*

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
$\{x: \ldots\}$	the set of all x such that
$\mathrm{n}(A)$	the number of elements in set A
$\emptyset$	the empty set
$\mathscr{E}$	universal set
A'	the complement of the set A
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$
$\mathbb{Q}_0^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geqslant 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R}: x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R}: x \geqslant 0\}$
$\mathbb{R}^n$	the real n-tuples
$\mathbb{C}$	the set of complex numbers
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\nsubseteq$	is not a subset of
$\not\subset$	is not a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leqslant x \leqslant b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leqslant x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leqslant b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$

2. *Miscellaneous Symbols*

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\propto$	is proportional to
$<$	is less than
$\leqslant$; $\ngtr$	is less than or equal to; is not greater than
$>$	is greater than
$\geqslant$; $\nless$	is greater than or equal to; is not less than
∞	infinity

3. *Operations*

$a + b$	a plus b
$a - b$	a minus b
$a \times b$, ab, $a.b$	a multiplied by b
$a \div b$, $\frac{a}{b}$, a/b	a divided by b
$a : b$	the ratio of a to b
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + ... + a_n$
$\sqrt{a}$	the positive square root of the real number a
$\lvert a \rvert$	the modulus of the real number a
$n!$	n factorial for $n \in \mathbb{Z}^+ \cup \{0\}$, $(0! = 1)$
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$, for $n, r \in \mathbb{Z}^+ \cup \{0\}$, $0 \leqslant r \leqslant n$ $\frac{n(n-1)...(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{Z}^+ \cup \{0\}$

4. *Functions*

f	the function f
f(x)	the value of the function f at x
f: $A \to B$	f is a function under which each element of set A has an image in set B
f: $x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
g ∘ f, gf	the composite function of f and g which is defined by (g ∘ f)(x) or gf(x) = g(f(x))
$\lim\limits_{x \to a} \mathrm{f}(x)$	the limit of f(x) as x tends to a
Δx; δx	an increment of x
$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
$\mathrm{f}'(x), \mathrm{f}''(x), \ldots, \mathrm{f}^{(n)}(x)$	the first, second, … nth derivatives of f(x) with respect to x
$\int y \,\mathrm{d}x$	indefinite integral of y with respect to x
$\int_a^b y \,\mathrm{d}x$	the definite integral of y with respect to x for values of x between a and b
$\dot{x}$, $\ddot{x}$, …	the first, second, …derivatives of x with respect to time

5. *Exponential and Logarithmic Functions*

e	base of natural logarithms
e^x, exp x	exponential function of x
$\log_a x$	logarithm to the base a of x
ln x	natural logarithm of x
lg x	logarithm of x to base 10

6. *Circular Functions and Relations*

sin, cos, tan, cosec, sec, cot	} the circular functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ cosec^{-1}, $\sec^{-1}$, $\cot^{-1}$	} the inverse circular functions

7. *Complex Numbers*

i	the square root of –1
z	a complex number, $z = x + \mathrm{i}y$ $= r(\cos\theta + \mathrm{i}\sin\theta),\ r \in \mathbb{R}_0^+$ $= r\mathrm{e}^{\mathrm{i}\theta},\ r \in \mathbb{R}_0^+$
Re z	the real part of z, Re $(x + \mathrm{i}y) = x$
Im z	the imaginary part of z, Im $(x + \mathrm{i}y) = y$
$\lvert z \rvert$	the modulus of z, $\lvert x + iy \rvert = \sqrt{x^2 + y^2}$, $\lvert r(\cos\theta + i\sin\theta) \rvert = r$
arg z	the argument of z, $\arg(r(\cos\theta + \mathrm{i}\sin\theta)) = \theta$, $-\pi < \theta \leqslant \pi$
z^*	the complex conjugate of z, $(x + \mathrm{i}y)^* = x - \mathrm{i}y$

8. *Matrices*

M	a matrix **M**
$\mathbf{M}^{-1}$	the inverse of the square matrix **M**
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix **M**
det **M**	the determinant of the square matrix **M**

9. *Vectors*

a	the vector **a**
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of the vector **a**
i, j, k	unit vectors in the directions of the Cartesian coordinate axes
$\lvert \mathbf{a} \rvert$	the magnitude of **a**
$\lvert \overrightarrow{AB} \rvert$	the magnitude of $\overrightarrow{AB}$
a.b	the scalar product of **a** and **b**
a×b	the vector product of **a** and **b**

10. *Probability and Statistics*

A, B, C, etc.	events
$A \cup B$	union of events A and B
$A \cap B$	intersection of the events A and B
$\mathrm{P}(A)$	probability of the event A
A'	complement of the event A, the event 'not A'
$\mathrm{P}(A \mid B)$	probability of the event A given the event B
X, Y, R, etc.	random variables
x, y, r, etc.	value of the random variables X, Y, R, etc.
x_1, x_2, …	observations
f_1, f_2,…	frequencies with which the observations, x_1, x_2 …occur
$\mathrm{p}(x)$	the value of the probability function $\mathrm{P}(X = x)$ of the discrete random variable X
p_1, p_2…	probabilities of the values x_1, x_2, …of the discrete random variable X
$\mathrm{f}(x)$, $\mathrm{g}(x)$…	the value of the probability density function of the continuous random variable X
$\mathrm{F}(x)$, $\mathrm{G}(x)$…	the value of the (cumulative) distribution function $\mathrm{P}(X \leqslant x)$ of the random variable X
$\mathrm{E}(X)$	expectation of the random variable X
$\mathrm{E}[\mathrm{g}(X)]$	expectation of $\mathrm{g}(X)$
$\mathrm{Var}(X)$	variance of the random variable X
$\mathrm{B}(n, p)$	binominal distribution, parameters n and p
$\mathrm{Po}(\mu)$	Poisson distribution, mean μ
$\mathrm{N}(\mu, \sigma^2)$	normal distribution, mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\overline{x}$	sample mean
s^2	unbiased estimate of population variance from a sample, $s^2 = \dfrac{1}{n-1}\sum\left(x - \overline{x}\right)^2$
ϕ	probability density function of the standardised normal variable with distribution N (0, 1)
Φ	corresponding cumulative distribution function
ρ	linear product-moment correlation coefficient for a population
r	linear product-moment correlation coefficient for a sample